Relaxation limit and initial layer to heat-conductive hydrodynamic models for semiconductor (Mathematical Analysis in Fluid and Gas Dynamics)

Suzuki, Masahiro; Nishibata, Shinya

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Relaxation limit and initial layer to heat-conductive hydrodynamic models for semiconductor

東京工業大学　鈴木 政尋 (Masahiro Suzuki)
東京工業大学　西畑 伸也 (Shinya Nishibata)

Department of Mathematical and Computing Sciences
Tokyo Institute of Technology, Tokyo 152-8552, Japan

1 Introduction

This short paper gives a brief survey on the results in the authors' previous papers [16, 17], which concern singular limits from a heat-conductive hydrodynamic model of semiconductors to an energy-transport and a drift-diffusion models. It also studies a singular limit from the energy-transport model to the drift-diffusion model. These limits are called "relaxation time limits" or "relaxation limits" in short. In order to discuss them, we study the existence and the asymptotic stability of a stationary solution to the above models. In all results of this paper, we do not need any smallness assumptions on the initial data provided that relaxation times are sufficiently small since the drift-diffusion model, a limited system, consists of a uniformly parabolic equation and the Poisson equation.

The heat-conductive hydrodynamic model is important to design the semiconductor device with a high performance since it accounts for a hot carrier problem, which is caused by high temperature and make the devices unstable. It is formulated as the system of equations, corresponding to a conservation low of mass, a balance low of momentum, a thermal equation and the Poisson equation,

$$\rho_s + m_x = 0, \tag{1.1a}$$

$$m_s + \left(\frac{m^2}{\rho} + p(\rho,\theta)\right)_x = \rho\phi_x - \frac{m}{\tau_m}, \tag{1.1b}$$

$$\rho\theta_s + m\theta_x + \frac{2}{3}\left(\frac{m}{\rho}\right)_x \rho\theta - \frac{2}{3}(\kappa\theta_x)_x = \frac{2\tau_e - \tau_m}{3\tau_m\tau_e}\frac{m^2}{\rho} - \frac{\rho}{\tau_e}(\theta - \bar{\vartheta}), \tag{1.1c}$$

$$\phi_{xx} = \rho - D. \tag{1.1d}$$

The unknown functions ρ, m, θ and ϕ denote electron density, electric current, absolute temperature and electrostatic potential, respectively. The pressure p is supposed to obey the Boyle-Charle law, that is,

$$p = p(\rho,\theta) = \rho\theta. \tag{1.2}$$

Positive constants $\bar{\theta}$, κ, τ_m and τ_e mean ambient device temperature, thermal conductivity, momentum relaxation time and energy relaxation time, respectively. The relaxation times τ_m and τ_e are small when electron density is dense in semiconductor. From the physical point of view, we assume $\tau_m \le \tau_e$. Moreover, $D(x)$ means doping profile, which is distribution of the density of positively ionized impurities in semiconductor devices. It is a given function in $\mathcal{B}^0(\overline{\Omega})$ and satisfies

$$\inf_{x\in\overline{\Omega}} D(x) > 0. \tag{1.3}$$

The system (1.1) is studied over the bounded domain $\Omega := (0,1)$ with the initial and the boundary conditions to the system (1.1) as

$$(\rho, m, \theta)(0,x) = (\rho_0, m_0, \theta_0)(x). \tag{1.4}$$

$$\rho(t,0) = \rho_l > 0, \quad \rho(t,1) = \rho_r > 0, \tag{1.5}$$

$$\theta_x(t,0) = \theta_x(t,1) = 0, \tag{1.6}$$

$$\phi(t,0) = 0, \quad \phi(t,1) = \phi_r \ge 0, \tag{1.7}$$

where ρ_l, ρ_r and ϕ_r are given constants. Integrating (1.1d) and using the boundary condition (1.7), we have an explicit formula of the electrostatic potential

$$\begin{aligned}\phi(t,x) &= \Phi[\rho](t,x)\\ &:= \int_0^x\int_0^y (\rho - D)(t,z)\,dzdy + \left(\phi_r - \int_0^1\int_0^y (\rho - D)(t,z)\,dzdy\right)x.\end{aligned} \tag{1.8}$$

To construct a classical solution, assume the compatibility condition hold at $(t,x) = (0,0)$ and $(t,x) = (0,1)$. Namely,

$$\rho_0(0) = \rho_l, \quad \rho_0(1) = \rho_r, \tag{1.9a}$$

$$\theta_{0x}(0) = \theta_{0x}(1) = 0, \tag{1.9b}$$

$$m_{0x}(0) = m_{0x}(1) = 0. \tag{1.9c}$$

Moreover the initial data are supposed to satisfy a subsonic condition and positivity of density and temperature

$$\inf_{x\in\Omega} \rho_0 > 0, \quad \inf_{x\in\Omega} \theta_0 > 0, \quad \inf_{x\in\Omega}\left(\theta_0 - \frac{m_0^2}{\rho_0^2}\right) > 0.$$

We construct the solution to problem (1.1) and (1.4)–(1.7) around the above initial data (ρ_0, j_0, θ_0) to satisfy same condition:

$$\inf_{x\in\Omega} \rho > 0, \tag{1.10a}$$

$$\inf_{x\in\Omega} \theta > 0, \tag{1.10b}$$

$$\inf_{x\in\Omega}\left(\theta - \frac{m^2}{\rho^2}\right) > 0. \tag{1.10c}$$

In Sections 3 and 4, we derive the drift-diffusion and the energy-transport models by the limit procedures to make relaxation times τ_m and/or τ_e. For this purpose, we introduce new variables

$$t := \frac{s}{\tau_m}, \quad j := \frac{m}{\tau_m}, \quad \varepsilon := \tau_m^2, \quad \zeta := \tau_m\tau_e, \quad \kappa_0 := \frac{\kappa}{\tau_m}$$

and substitute these in (1.1). Then the heat-conductive hydrodynamic model is rewritten as

$$\rho_t + j_x = 0, \tag{1.11a}$$

$$\varepsilon j_t + \left(\varepsilon\frac{j^2}{\rho} + \rho\theta\right)_x = \rho\phi_x - j, \tag{1.11b}$$

$$\rho\theta_t + j\theta_x + \frac{2}{3}\left(\frac{j}{\rho}\right)_x \rho\theta - \frac{2}{3}\kappa_0\theta_{xx} = \left(\frac{2}{3} - \frac{\varepsilon}{3\zeta}\right)\frac{j^2}{\rho} - \frac{\rho}{\zeta}(\theta - \bar{\vartheta}), \tag{1.11c}$$

$$\phi_{xx} = \rho - D. \tag{1.11d}$$

The initial data to (1.11) follows from (1.12a):

$$\rho(0,x) = \rho_0(x), \tag{1.12a}$$

$$j(0,x) = j_0(x) := m_0/\tau_m(x), \tag{1.12b}$$

$$\theta(0,x) = \theta_0(x). \tag{1.12c}$$

The boundary data to (1.11) is given by (1.5)–(1.7). The subsonic condition (1.10c) is rewritten as

$$\inf_{x\in\Omega}\left(\theta - \varepsilon\frac{j^2}{\rho^2}\right) > 0, \tag{1.13}$$

which follows from the positivity of temperature (1.10b) for the suitably small momentum relaxation $\tau_m = \sqrt{\varepsilon}$. The time global solvability of the above initial boundary value problem is studied in Section 2.

Notation. For a nonnegative integer $l \geq 0$, $H^l(\Omega)$ denotes the l-th order Sobolev space in the L^2 sense, equipped with the norm $\|\cdot\|_l$. We note $H^0 = L^2$ and $\|\cdot\| := \|\cdot\|_0$. $C^k([0,T]; H^l(\Omega))$ denotes the space of the k-times continuously differentiable functions on the interval $[0,T]$ with values in $H^l(\Omega)$. $H^k(0,T;H^l(\Omega))$ is the space of H^k–functions on $(0,T)$ with values in $H^l(\Omega)$. Moreover, $\mathfrak{X}$, $\mathfrak{Y}$ and $\mathfrak{Z}$ denote the function spaces

$$\mathfrak{X}_i^j([0,T]) := \bigcap_{k=0}^{i} C^k([0,T]; H^{j+i-k}(\Omega)), \quad \mathfrak{X}_i([0,T]) := \mathfrak{X}_i^0([0,T]),$$

$$\mathfrak{Y}([0,T]) := C^1([0,T]; L^2(\Omega)) \cap C^0([0,T]; H^2(\Omega)) \cap H^1(0,T; H^1(\Omega)),$$

$$\mathfrak{Z}([0,T]) := C([0,T]; H^1(\Omega)) \cap L^2(0,T; H^2(\Omega)) \cap H^1(0,T; L^2(\Omega))$$

for nonnegative integers $i, j \geq 0$, where $[\mu]$ denotes the largest integer which is less than or equal to μ. For a nonnegative integer $k \geq 0$, $\mathcal{B}^k(\overline{\Omega})$ denotes the space of the functions whose derivatives up to k-th order are continuous and bounded over $\overline{\Omega}$.

2 Heat-conductive hydrodynamic model

This section is devoted to considering the unique existence and the asymptotic stability of a stationary solution $(\tilde{\rho}, \tilde{j}, \theta, \tilde{\phi})$ to the heat-conductive hydrodynamic model (1.11). Since the stationary solution is a solution independent of a time variable t, it satisfies a system of equations

$$\tilde{j}_x = 0, \tag{2.1a}$$
$$S[\tilde{\rho}, \tilde{j}, \tilde{\theta}]\tilde{\rho}_x + \tilde{\rho}\tilde{\theta}_x = \tilde{\rho}\tilde{\phi}_x - \tilde{j}, \tag{2.1b}$$
$$\tilde{j}\tilde{\theta}_x - \frac{2}{3}\tilde{j}\tilde{\theta}\,(\log\tilde{\rho})_x - \frac{2}{3}\kappa_0\tilde{\theta}_{xx} = \left(\frac{2}{3} - \frac{\varepsilon}{3\zeta}\right)\frac{\tilde{j}^2}{\tilde{\rho}} - \frac{\tilde{\rho}}{\zeta}(\tilde{\theta} - \bar{\vartheta}), \tag{2.1c}$$
$$\tilde{\phi}_{xx} = \tilde{\rho} - D \tag{2.1d}$$

with boundary condition (1.5)–(1.7). Here the strength of the boundary data

$$\delta := |\rho_r - \rho_l| + |\phi_r|$$

plays a crucial role in the discussion about the unique existence and the asymptotic stability of the stationary solution.

Many mathematicians study the unique existence and the asymptotic stability of the stationary solution to the isentropic and the isothermal hydrodynamic models (see [5, 7, 9, 10, 13]). In particular, the papers [7, 13] show the stability theorem with non-flat doping profile. Recently, the authors in [15] extend the results in [7, 13] to the heat-conductive hydrodynamic model. Precisely, we discuss the unique existence and the asymptotic stability of the stationary solution to this model with the Dirichlet boundary condition $\theta(t,0) = \theta_l$ and $\theta(t,0) = \theta_l$ in place the Neumann boundary condition (1.6). The above stability results require the smallness of the initial disturbance. In [17], we succeed in showing the stability theorem to the heat-conductive hydrodynamic model without any smallness assumption on the initial disturbance. Instead of it, the parameters ε and ζ are required to be suitably small subject to the initial data. This result is summarized in Theorem 2.2. We state the unique existence of stationary solution in Lemma 2.1.

Lemma 2.1. *Let the doping profile and the boundary data satisfy conditions* (1.3), (1.5) *and* (1.7). *For an arbitrary* ρ_l, *there exits a positive constant* δ_0, ζ_0 *and* η_0 *such that if* $\delta \leq \delta_0$ *and* $\varepsilon < \zeta \leq \zeta_0$, *then there exists a unique stationary solution* $(\tilde{\rho}, \tilde{j}, \tilde{\theta}, \tilde{\phi}) \in \mathcal{B}^2(\overline{\Omega}) \times \mathcal{B}^2(\overline{\Omega}) \times \mathcal{B}^3(\overline{\Omega}) \times \mathcal{B}^2(\overline{\Omega})$ *to* (2.1) *and* (1.5)–(1.7), *satisfying the conditions* (1.10c),

$$\frac{1}{2}B_m \leq \tilde{\rho} \leq 2B_M, \tag{2.2a}$$
$$B_m := \min\left\{\rho_l,\ \rho_r,\ \inf_{x\in\Omega} D(x)\right\}, \quad B_M := \max\left\{\rho_l,\ \rho_r,\ \sup_{x\in\Omega} D(x)\right\},$$
$$\sup_{x\in\overline{\Omega}} |\tilde{\theta} - \bar{\vartheta}| \leq \eta_0. \tag{2.2b}$$

Theorem 2.2. *Let $(\tilde{\rho}, \tilde{j}, \tilde{\theta}, \tilde{\phi})$ be the stationary solution to the hydrodynamic model* (2.1). *Suppose that the initial data $(\rho_0, j_0, \theta_0) \in H^2(\Omega) \times H^2(\Omega) \times H^3(\Omega)$ and the boundary data ρ_l, ρ_r and ϕ_r satisfy the conditions* (1.5), (1.7), (1.9), (1.10a), (1.10b) *and* (1.13). *Then there exist positive constants δ_0 and ζ_0 such that if $\delta \leq \delta_0$ and $\zeta \leq \zeta_0$, there exist a positive constant ε_0, depending on ζ but independent of δ, such that if $\varepsilon \leq \varepsilon_0$, the initial boundary value problem* (1.5)–(1.7), (1.11) *and* (1.12) *has a unique solution (ρ, j, θ, ϕ) satisfying $\rho - \tilde{\rho}$, $j - \tilde{j} \in \mathfrak{X}_2([0,\infty))$, $\theta - \tilde{\theta}, \theta_x - \tilde{\theta}_x \in \mathfrak{Y}([0,\infty))$, $\phi - \tilde{\phi} \in \mathfrak{X}_2^2([0,\infty))$ and the conditions* (1.10a), (1.10b) *and* (1.13). *Moreover, the solution (ρ, j, θ, ϕ) verifies the decay estimate*

$$
\begin{aligned}
&\|(j - \tilde{j})(t)\|_1^2 + \|(\rho - \tilde{\rho}, \theta - \tilde{\theta})(t)\|_2^2 \\
&\qquad + \varepsilon\|(\partial_x^2\{j - \tilde{j}\}, \partial_x^3\{\theta - \tilde{\theta}\})(t)\|^2 + \|(\phi - \tilde{\phi})(t)\|_4^2 \leq Ce^{-\alpha t},
\end{aligned}
$$

where C and α are positive constants depending on ζ but independent of δ, ε and t.

3 Energy-transport model

The energy-transport model is formally obtained by letting the parameter ε to zero in (1.11):

$$\rho_t + j_x = 0, \tag{3.1a}$$

$$\rho\theta_t + j\theta_x + \frac{2}{3}\left(\frac{j}{\rho}\right)_x \rho\theta - \frac{2}{3}\kappa_0\theta_{xx} = \frac{2}{3}\frac{j^2}{\rho} - \frac{\rho}{\zeta}(\theta - \bar{\vartheta}), \tag{3.1b}$$

$$\phi_{xx} = \rho - D, \tag{3.1c}$$

where the electric current is given by

$$j = \rho\phi_x - (\theta\rho)_x. \tag{3.1d}$$

We prescribe the initial and the boundary data (1.12a) and (1.12c) as well as (1.5)–(1.7), respectively. This section states justification of this limit procedure rigorously. Precisely, it is discussed that the solution to the heat-conductive hydrodynamic model (1.11) converges to that to the energy-transport model (3.1) as ε tends to zero.

Time global solvability. We introduce the results concerning the time global solvability to the energy-transport model (3.1). Degond, Génieys and Jüngel [3, 4] analyze the general parabolic system coupled with the Poisson equation, which covers the energy-transport model, over the multi-dimensional bounded domain with the Dirichlet-Neumann mixed boundary conditions. They prove the unique existence and the asymptotic stability of the stationary solution. These results, however, assume uniform parabolicity. We in [17] obtain the same result without assuming the uniform parabolicity even only for the one-dimensional problem. These results are summarized in Lemma 3.1 and Theorem 3.2.

Lemma 3.1. *Let the doping profile and the boundary data satisfy conditions* (1.3), (1.5) *and* (1.7). *For an arbitrary* ρ_l*, there exits a positive constant* δ_0*,* ζ_0 *and* η_0 *such that if* $\delta \le \delta_0$ *and* $\zeta \le \zeta_0$*, then there exists a unique stationary solution* $(\tilde{\rho}, \tilde{j}, \tilde{\theta}, \tilde{\phi}) \in \mathcal{B}^2(\overline{\Omega}) \times \mathcal{B}^2(\overline{\Omega}) \times \mathcal{B}^3(\overline{\Omega}) \times \mathcal{B}^2(\overline{\Omega})$ *to the energy-transport model* (3.1), *satisfying the condition* (2.2).

Theorem 3.2. *Let* $(\tilde{\rho}, \tilde{j}, \tilde{\theta}, \tilde{\phi})$ *be the stationary solution to the energy-transport model* (3.1). *Suppose that the initial data* $(\rho_0, \theta_0) \in H^1(\Omega)$ *and the boundary data* ρ_l*,* ρ_r *and* ϕ_r *satisfy* (1.5), (1.7), (1.9a), (1.10a) *and* (1.10b). *Then there exist positive constants* δ_0 *and* ζ_0 *such that if* $\delta \le \delta_0$ *and* $\zeta \le \zeta_0$*, the initial boundary value problem* (1.5)-(1.7), (1.12a), (1.12c) *and* (3.1) *has a unique solution* (ρ, j, θ, ϕ) *satisfying* $\rho - \tilde{\rho}, \theta - \tilde{\theta} \in \mathfrak{Z}([0,\infty)) \cap \mathfrak{Y}_{loc}((0,\infty))$, $j - \tilde{j} \in C([0,\infty); L^2(\Omega)) \cap \mathfrak{Z}_{loc}((0,\infty))$, $\phi - \tilde{\phi} \in C([0,\infty); H^3(\Omega)) \cap H^1([0,\infty); H^2(\Omega))$*; the positivity* (1.10a) *and* (1.10b). *Moreover, it verifies* $\sqrt{t}\rho_{xt}, \sqrt{t}\theta_{xt} \in L^2(0,\infty : L^2(\Omega))$ *and the decay estimates*

$$\|(j - \tilde{j})(t)\|^2 + \|(\rho - \tilde{\rho}, \theta - \tilde{\theta})(t)\|_1^2 + \|(\phi - \tilde{\phi})(t)\|_3^2 \le Ce^{-\alpha t},$$
$$t\|(j_x - \tilde{j}_x)(t)\|^2 + \frac{t}{\zeta}\|(\theta - \tilde{\theta})(t)\|_1^2 + t\|(\rho - \tilde{\rho}, \theta - \tilde{\theta})(t)\|_2^2 \le Ce^{-\alpha t},$$

where C *and* α *are positive constants independent of* ζ*,* δ *and* t*.*

Relaxation limit. For the full space $\mathbb{R}$, the relaxation limit from the hydrodynamic model to the energy-transport model is justified by Ali, Bini and Rionero [1]. They assume that the initial data is close enough to the special stationary solution, $\tilde{j} = 0$ and $\tilde{\theta} = \vartheta$. We justify this relaxation limit for the large initial data under the physically admissible boundary condition.

Theorem 3.3. *Suppose that the initial data* $(\rho_0, j_0, \theta_0) \in H^2(\Omega) \times H^2(\Omega) \times H^3(\Omega)$ *and the boundary data* ρ_l*,* ρ_r *and* ϕ_r *satisfy the conditions* (1.5), (1.7), (1.9), (1.10a), (1.10b) *and* (1.13). *Then there exist positive constants* δ_0 *and* ζ_0 *such that if* $\delta \le \delta_0$ *and* $\zeta \le \zeta_0$*, there exists a positive constant* ε_0*, depending on* ζ *but independent of* δ*, such that if* $\varepsilon \le \varepsilon_0$*, the time global solution* $(\rho_\zeta^\varepsilon, j_\zeta^\varepsilon, \theta_\zeta^\varepsilon, \phi_\zeta^\varepsilon)$ *for the problem* (1.5)-(1.7), (1.11) *and* (1.12) *converges to the time global solution* $(\rho_\zeta^0, j_\zeta^0, \theta_\zeta^0, \phi_\zeta^0)$ *for the problem* (1.5)-(1.7), (1.12a), (1.12c) *and* (3.1) *as* ε *tends to zero. Precisely, the following estimates hold for an arbitrary* $t \in (0,\infty)$*.*

$$\|(\rho_\zeta^\varepsilon - \rho_\zeta^0, \theta_\zeta^\varepsilon - \theta_\zeta^0)(t)\|_1^2 + \|(\phi_\zeta^\varepsilon - \phi_\zeta^0)(t)\|_3^2 \le C\varepsilon^\gamma, \tag{3.2}$$
$$\|(j_\zeta^\varepsilon - j_\zeta^0)(t)\|^2 \le \|(j_0 - j_\zeta^0)(0)\|^2 e^{-t/\varepsilon} + C\varepsilon^\gamma, \tag{3.3}$$
$$\|(\partial_x^2\{\rho_\zeta^\varepsilon - \rho_\zeta^0\}, \partial_x^1\{j_\zeta^\varepsilon - j_\zeta^0\}, \partial_x^2\{\theta_\zeta^\varepsilon - \theta_\zeta^0\}, \partial_x^4\{\phi_\zeta^\varepsilon - \phi_\zeta^0\})(t)\|^2 \le C\varepsilon^\gamma(t^{-2} + 1), \tag{3.4}$$

where γ *and* C *are positive constants depending on* ζ *but independent of* ε*,* δ *and* t*.*

Remark 3.4. *Three initial conditions in* (1.12) *are required for* (1.11) *while we prescribe two initial conditions for* (3.1) *only. On the other hand, the initial value* $j^0_\zeta(0,x)$ *to* (3.1) *is explicitly written as*

$$j^0_\zeta(0,x) = (-(\theta_0\rho_0)_x + \rho_0\{\Phi[\rho_0]\}_x)(x). \tag{3.5}$$

thanks to (1.8) *and* (3.1d). *Here it is not necessary that the initial data* $j_0(x)$ *equals* (3.5). *Hence, in the justification of the relaxation limits, the difference between* $j_0(x) - j^0_\zeta(0,x)$ *gives rise to the initial layer. However, by the estimate* (3.3), *the layer is shown to decay as time* t *tends to infinity and/or the parameter* ε *tends to zero.*

4 Drift-diffusion model

Letting the parameter ζ tend to zero in (3.1) or letting the parameters ε and ζ tend to zero in (1.11), we have the drift-diffusion model

$$\rho_t + j_x = 0, \tag{4.1a}$$
$$\phi_{xx} = \rho - D, \tag{4.1b}$$

where the electric current is given by

$$j = \rho\phi_x - (\bar{\vartheta}\rho)_x. \tag{4.1c}$$

The initial and the boundary data to (4.1) are (1.12a) as well as (1.5) and (1.7), respectively. We state the results on rigorous justification of the above two limits. To this end, we begin brief discussion about the time global solvability of the drift-diffusion model (4.1).

Time global solvability. Mock [11, 12] proves the existence and the asymptotic stability of the stationary solution over the multi-dimensional bounded domain with a special boundary condition, which does not allow any electron flow through the boundary. After that, Gajewski and Gröger in [6] obtain the same results in adopting more general boundary conditions, which covers the case electric current permeates the boundary. However, the research in [6] analyze the special stationary solution, in which no electron flows. We extend it to the general stationary solution even only over the one-dimensional bounded domain. These results are summarized in the next two lemmas.

Lemma 4.1. *Let the doping profile and the boundary data satisfy conditions* (1.3), (1.5) *and* (1.7). *Then there exists a unique stationary solution* $(\tilde{\rho}, \tilde{j}, \tilde{\phi}) \in \mathcal{B}^2(\overline{\Omega})$ *to the drift-diffusion* (4.1), *satisfying the condition* (1.10a).

Lemma 4.2. *Let $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$ be the stationary solution to the drift-diffusion* (4.1). *Suppose that the initial data $\rho_0 \in H^1(\Omega)$ and the boundary data ρ_l, ρ_r and ϕ_r satisfy* (1.5), (1.7), (1.9a) *and* (1.10a). *Then there exists a positive constant δ_0 such that if $\delta \leq \delta_0$, the initial boundary value problem* (1.5), (1.7), (1.12a) *and* (4.1) *has a unique solution (ρ, j, ϕ) satisfying $\rho - \tilde{\rho} \in \mathfrak{Z}([0,\infty)) \cap \mathfrak{Y}_{loc}((0,\infty))$, $j - \tilde{j} \in C([0,\infty); L^2(\Omega)) \cap \mathfrak{Y}([0,\infty))$, $\phi - \tilde{\phi} \in C([0,\infty); H^3(\Omega)) \cap H^1(0,\infty; H^2(\Omega))$ and the positivity* (1.10a). *Moreover it verifies the estimate*

$$\|(\rho - \tilde{\rho})(t)\|_1^2 + \|(j - \tilde{j})(t)\|^2 + \|(\phi - \tilde{\phi})(t)\|_3^2 \leq Ce^{-\alpha t},$$

where C and α are positive constants independent of t and δ.

Relaxation limit. Chen, Jerome and Zhang [2] justify the relaxation limit from the hydrodynamic model to the drift-diffusion model with the flat doping profile and the Dirichlet zero boundary condition on electric current, which makes no current $\tilde{j} = 0$. However the typical doping profile have steep slops. Thus we study this limit under the physically admissible conditions. The result is summarized in Corollary 4.5. The corollary immediately follows from Theorem 3.3, once we show the next theorem, concerning the relaxation limit from the energy-transport model (3.1) to the drift-diffusion model (4.1).

Theorem 4.3. *Suppose that the initial data $(\rho_0, \theta_0) \in H^1(\Omega)$ and the boundary data ρ_l, ρ_r and ϕ_r satisfy* (1.5), (1.7), (1.9a), (1.10a) *and* (1.10b). *Then there exist positive constants δ_0 and ζ_0 such that if $\delta \leq \delta_0$ and $\zeta \leq \zeta_0$, then the time global solution $(\rho_\zeta^0, j_\zeta^0, \theta_\zeta^0, \phi_\zeta^0)$ for the problem* (1.5)–(1.7), (1.12a), (1.12c) *and* (3.1) *converges to the time global solution $(\rho_0^0, j_0^0, \phi_0^0)$ for the problem* (1.5), (1.7), (1.12a) *and* (4.1) *as ζ tends to zero. Precisely, the following estimates hold for an arbitrary $t \in [0,\infty)$.*

$$\|(\rho_\zeta^0 - \rho_0^0)(t)\|^2 + \|(\phi_\zeta^0 - \phi_0^0)(t)\|_2^2 \leq C\zeta^\gamma, \tag{4.2}$$

$$\|(\theta_\zeta^0 - \bar{\vartheta})(t)\|^2 \leq C\|\theta_0 - \bar{\vartheta}\|^2 e^{-\nu t/\zeta} + C\zeta^\gamma, \tag{4.3}$$

$$\|(\{\rho_\zeta^0 - \rho_0^0\}_x, \{\theta_\zeta^0\}_x, j_\zeta^0 - j_0^0)(t)\|^2 \leq C\zeta^\gamma(1 + t^{-1}), \tag{4.4}$$

where ν, γ and C are positive constants independent of ζ, δ and t.

Remark 4.4. *As we can not impose the initial data* (1.12c) *for* (4.1), *the difference $\theta_0(x) - \bar{\vartheta}$ gives rise to the initial layer similarly as in the relaxation limit from* (1.11) *to* (3.1). *The estimate* (4.3) *shows the layer decays as time t tends to infinity and/or the parameter ζ tends to zero.*

Theorems 3.3 and 4.3 immediately give the next corollary, which asserts the justification of the relaxation limit from the hydrodynamic model (1.11) to the drift-diffusion model (4.1).

Corollary 4.5. *Assume the same conditions as in Theorems 3.3 and 4.3. Then the time global solution* $(\rho^\varepsilon_\zeta, j^\varepsilon_\zeta, \theta^\varepsilon_\zeta, \phi^\varepsilon_\zeta)$ *for the problem* (1.5)–(1.7), (1.11) *and* (1.12) *converges to the time global solution* $(\rho^0_0, j^0_0, \bar{\vartheta}, \phi^0_0)$ *for the problem* (1.5), (1.7), (1.12a) *and* (4.1) *as* ε *tends to zero. Precisely, the following estimates hold for an arbitrary* $t \in (0, \infty)$.

$$\|(\rho^\varepsilon_\zeta - \rho^0_0)(t)\|^2 + \|(\phi^\varepsilon_\zeta - \phi^0_0)(t)\|_2^2 \le \overline{C}\zeta^{\overline{\gamma}} + C\varepsilon^\gamma,$$
$$\|(\theta^\varepsilon_\zeta - \bar{\vartheta})(t)\|^2 \le \overline{C}\|\theta_0 - \bar{\vartheta}\|^2 e^{-\overline{\nu}t/\zeta} + \overline{C}\zeta^{\overline{\gamma}} + C\varepsilon^\gamma,$$
$$\|(j^\varepsilon_\zeta - j^0_0)(t)\|^2 \le \overline{C}\|(j_0 - j^0_0)(0)\|^2 e^{-t/\varepsilon} + \overline{C}\zeta^{\overline{\gamma}}(1 + t^{-1}) + C\varepsilon^\gamma,$$
$$\|(\{\rho^\varepsilon_\zeta - \rho^0_0\}_x, \{\theta^\varepsilon_\zeta\}_x)(t)\|^2 \le \overline{C}\zeta^{\overline{\gamma}}(1 + t^{-1}) + C\varepsilon^\gamma.$$

Here γ *and* C *are positive constants depending on* ζ *but independent of* ε, δ *and* t. *Also* $\overline{\nu}$, $\overline{\gamma}$ *and* $\overline{C}$ *are positive constants independent of* ε, ζ, δ *and* t.

5 Isothermal hydrodynamic model

By assuming the temperature θ is constant in (1.1), we have the isothermal hydrodynamic model

$$\rho_t + j_x = 0, \tag{5.1a}$$
$$\varepsilon j_t + \left(\varepsilon \frac{j^2}{\rho} + K\rho\right)_x = \rho\phi_x - j, \tag{5.1b}$$
$$\phi_{xx} = \rho - D. \tag{5.1c}$$

The initial and boundary data are (1.12a) and (1.12b) as well as (1.5) and (1.7), respectively. This section gives the brief survey on the results in [16]. Precisely, we study the relaxation limit from the isothermal hydrodynamic model (5.1) to the drift-diffusion model (4.1).

Time global solvability. The asymptotic stability of the stationary solution are shown with the non-flat doping profile in [7, 13]. These two results assume the smallness of the initial disturbance. We obtain the same result without any smallness assumption on the initial disturbance provided that the parameter ε is sufficiently small subject to initial data. It is summarized in Theorem 5.2. The unique existence of the stationary solution is stated in Lemma 5.1.

Lemma 5.1. *Let the doping profile and the boundary data satisfy conditions* (1.3), (1.5) *and* (1.7). *Then there exists a positive constant* δ_0 *such that if* $\delta \le \delta_0$, *the isothermal hydrodynamic model has a unique stationary solution* $(\tilde{\rho}, \tilde{j}, \tilde{\phi}) \in \mathcal{B}^2(\overline{\Omega})$ *satisfying the conditions* (1.10a) *and* $\inf(K - \varepsilon\tilde{j}^2/\tilde{\rho}^2) > 0$.

Theorem 5.2. *Let $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$ be the stationary solution to the isothermal hydrodynamic model* (5.1). *Suppose that the initial data $(\rho_0, j_0) \in H^2(\Omega)$ and the boundary data ρ_l, ρ_r and ϕ_r satisfy* (1.5), (1.7), (1.9a), (1.9c), (1.10a) *and* $\inf(K - \varepsilon\tilde{j}^2/\tilde{\rho}^2) > 0$. *Then there exist positive constants δ_0 and ε_0 such that if $\delta \leq \delta_0$ and $\varepsilon \leq \varepsilon_0$, the initial boundary value problem* (1.5), (1.7), (1.12a), (1.12b) *and* (5.1) *has a unique solution (ρ, j, ϕ) in the space $\mathfrak{X}_2([0, \infty))$ satisfying* (1.10a) *and* $\inf(K - \varepsilon j^2/ro^2) > 0$. *Moreover, the solution (ρ, j, ϕ) verifies the additional regularity $\phi - \tilde{\phi} \in \mathfrak{X}_4^2([0, \infty))$ and the decay estimate*

$$\|(\rho - \tilde{\rho})(t)\|_2^2 + \|(j - \tilde{j})(t)\|_1^2 + \|\sqrt{\varepsilon}(j - \tilde{j})_{xx}(t)\|^2 + \|(\phi - \tilde{\phi})(t)\|_4^2 \leq Ce^{-\alpha t},$$

where C and α are positive constants independent of t, δ and ε.

Relaxation limit. The relaxation limit from the isothermal hydrodynamic model to the drift-diffusion model is often investigated (for example see [2, 8, 16, 18]). Especially, we in [16] justify this limit of the solution with the initial layer for the large initial data.

Theorem 5.3. *Assume same conditions as in Theorem 5.2. Then there exist positive constants δ_0 and ε_0 such that if $\delta \leq \delta_0$ and $\varepsilon \leq \varepsilon_0$, then the time global solution $(\rho^\varepsilon, j^\varepsilon, \phi^\varepsilon)$ for* (1.5), (1.7), (1.12a), (1.12b) *and* (5.1) *converges to the time global solution (ρ^0, j^0, ϕ^0) for* (1.5), (1.7), (1.12a) *and* (4.1) *as ε tends to zero. Precisely, there exists a positive constant γ such that the following estimates hold.*

$$\begin{gathered}
\|(\rho^\varepsilon - \rho^0)(t)\|_1^2 + \|(\phi^\varepsilon - \phi^0)(t)\|_3^2 \leq C\varepsilon^\gamma, \\
\|(j^\varepsilon - j^0)(t)\|^2 \leq \|(j^\varepsilon - j^0)(0)\|^2 e^{-t/\varepsilon} + C\varepsilon^\gamma, \\
\|(\partial_x^2\{\rho^\varepsilon - \rho^0\}, \partial_x^1\{j^\varepsilon - j^0\}, \partial_x^4\{\phi^\varepsilon - \phi^0\})(t)\|^2 \leq C\varepsilon^\gamma(t^{-1} + 1)
\end{gathered}$$

for an arbitrary $t \in (0, \infty)$, where C is a positive constant independent of ε, δ and t.

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